(2.4), (2.6), we conclude that t, U is a solution of the problem (1.13), (1.14), (2.1).

Let us note the following:

1) The proof of the convergence of the proposed method of solving the Cauchy problem is carried over to the case of the characteristic problem as well as the mixed problem for the system (1.7) without essential change since it has been carried out by the method of characteristics.

2) The method considered for the solution of the incomplete plasticity equations can be applied to arbitrary hyperbolic quasi-linear systems with two independent variables admitting of separation in the above-mentioned sense.

3) The approximate method presented for solving the incomplete plasticity equations corresponding to the faces of the Tresca prism for the axisymmetric case reduces essentially to solving a number of plane problems of ideal plasticity theory (plane strain), whose numerical solution methods are quite well developed; the difference from the plane problem will consist only in the presence of an inhomogeneity in the equations under consideration (see (1.10), (1.11)).

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REFERENCES

1. Khristianovich, S. A. and Shemiakin, E. I., On the theory of ideal plasticity. Inzh. Zh,. Mekhan. Tverd. Tela, №4, 1967.
2. Haar, A. and Kármán, Th., Zur Theorie der Spannungszustände in plastischen und sandartigen Medien. Nachr. von der Königlichen Gesellschaft der Wissenschaft zu Göttingen. Math.-phys., Kl, p.204, 1909.
3. Rozhdestvenskii, B. L. and Ianenko, N. N., System of Quasi-linear Equations and their Application to Gasdynamics. "Nauka", Moscow, 1968.
4. Courant, R., Partial Differential Equations. New York, Interscience, 1962.

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ON PLASTIC INSTABILITY IN SOME CASES OF SIMPLE FLOW

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The stability of deformation of an elastic viscoplastic hardening material under high precritical strains is investigated in a three-dimensional formulation. A solution of the stability equations is obtained in a rectangular coordinate system for a developed fundamental plastic flow process with small elastic strains in the case of a homogeneous precritical state. The surface and internal instability phenomena are investigated.

The papers [1, 2] are devoted to an investigation of the stability of deformation of an elastic-plastic material with large precritical strains. The stability of deformation of bodies of viscoplastic and elastic-viscoplastic material under small precritical strains has been considered in a three-dimensional formulation in [3 – 7].

1. Let us describe the motion of a deformable medium by the equations [8]

$$x^\alpha = x^\alpha(X^i, t), \qquad X^i = X^i(x^\alpha, t) \tag{1.1}$$

The symbol x^α denotes a fixed coordinate system relative to which the motion of the medium occurs; X^i is a moving system of convective coordinates with the metric tensor g_{ij}.

Let us isolate three positions of the accompanying coordinates: initial corresponding to the absence of stresses, fully deformed, and new initial position with residual plastic strains in which internal stresses are absent. Let g°_{ij}, $g\hat{}_{ij}$ and g^*_{ij} denote the metric tensors, respectively. The space corresponding to the metric tensor g_{ij}^*, is generally non-Euclidean. Let us henceforth limit ourselves to the case of a homogeneous stress-strain precritical state in the body, which will permit remaining within the framework of Euclidean space.

We consider an elastic viscoplastic body, whose mechanical model is presented in [6]. The relationships between the stress and strain in this body can be written thus.

The body remains elastic while

$$s\hat{}^i_j s\hat{}^j_i < k^2(0), \qquad s_j^i = \sigma_j^i - {}^1/_3\sigma_n{}^n g_j{}^i \tag{1.2}$$

If $s\hat{}^i_j s\hat{}^j_i \geqslant k^2(\varkappa)$, $\varkappa = (s^{*i}_j - ce^{*pi}_j)\, e^{*pj}_i$, $e_j{}^i = \varepsilon_j{}^i - {}^1/_3\varepsilon_n{}^n g_j{}^i$, then

$$\hat{\varepsilon}_{ij} = \hat{\varepsilon}_{ij}{}^e + \varepsilon^{*p}_{ij} \tag{1.3}$$

The elastic strains are hence related to the stresses by Hooke's law

$$\sigma\hat{}^i_j = \lambda\varepsilon\hat{}^{en}_n g^i_j + 2\mu\varepsilon\hat{}^{ei}_j \tag{1.4}$$

The plastic strain rates are

$$\xi^{*pi}_j = 0, \qquad (s^{*i}_j - ce^{*pi}_j)(s^{*j}_i - ce^{*pj}_i) < k^2(\varkappa) \tag{1.5}$$

$$\xi^{*pi}_j = \psi(s^{*i}_j - ce^{*pi}_j - \eta\xi^{*pi}_j), \quad (s^{*i}_j - ce^{*pi}_j - \eta\xi^{*pi}_j)(s^{*j}_i - ce^{*pj}_i - \eta\xi^{*pj}_i) = k^2(\varkappa)$$

For the components with mixed structure of the indices, (1.3) can be represented as

$$\varepsilon\hat{}^i_j = \varepsilon\hat{}^{ei}_j + \varepsilon^{*pi}_j - 2\varepsilon^{*pn}_j\varepsilon\hat{}^{ei}_n \tag{1.6}$$

In this case, the equality

$$\xi\hat{}^i_j = \xi\hat{}^{ei}_j + \xi^{*pi}_j - 2\xi^{*pn}_j\varepsilon\hat{}^{ei}_n \tag{1.7}$$

is valid. Here $\xi\hat{}^i_j$, $\xi\hat{}^{ei}_j$, ξ^{*pi}_j are mixed velocity components of the full, elastic and plastic strains, respectively, in the bases $g\hat{}_{ij}$, g^*_{ij}. On the basis of Hooke's law the elastic strain rates satisfy the equation

$$\Sigma\hat{}^i_j = \lambda\xi\hat{}^{en}_n g^i_j + 2\mu\xi\hat{}^{ei}_j \tag{1.8}$$

$$\left(\Sigma^i_j = \frac{\partial}{\partial t}\sigma^i_j + \xi^i_n\sigma^n_j - \sigma^i_n\xi^n_j\right)$$

Here Σ^i_j is the Jaumann derivative of the stress tensor in the concomitant coordinate system [9]. Superposing a small additional motion determined by the displacement vector $\gamma \mathbf{w}'(X^i, t)$ (γ is a small parameter) on the main motion described by the relationships (1.1) – (1.8), let us write the linearized equations. We have, to the accuracy of linear terms, in the plastic domain

$$\sigma'^{\wedge i}_{\ j} = \lambda\varepsilon'^{\wedge en}_{\ \ n} g^i_j + 2\mu\varepsilon'^{\wedge ei}_{\ \ j}, \qquad \Sigma'^{\wedge i}_{\ j} = \lambda\xi'^{\wedge en}_{\ \ n} g^i_j + 2\mu\xi'^{\wedge ei}_{\ \ j} \tag{1.9}$$

$$\varepsilon'^{\wedge i}_{\ j} = \varepsilon'^{\wedge ei}_{\ \ j} + \varepsilon'^{*pi}_{\ \ j} - 2\varepsilon'^{*pn}_{\ \ j}\varepsilon^{\wedge ei}_{\ \ n} - 2\varepsilon^{*pn}_{\ \ j}\varepsilon'^{\wedge ei}_{\ \ n}$$

$$\xi'^{\wedge i}_{\ j} = \xi'^{\wedge ei}_{\ \ j} + \xi'^{*pi}_{\ \ j} - 2\xi'^{*pn}_{\ \ j}\varepsilon^{\wedge ei}_{\ \ n} - 2\xi^{*pn}_{\ \ j}\varepsilon'^{\wedge ei}_{\ \ n}$$

$$\xi'^{*pi}_{\ \ j} = \psi(s'^{*i}_{\ \ j} - ce'^{*pi}_{\ \ j} - \eta\xi'^{*pi}_{\ \ j}) + \psi'(s^{*i}_{\ j} - ce^{*pi}_{\ \ j} - \eta\xi^{*pi}_{\ \ j})$$

$$(s^{*i}_{\ j} - ce^{*pi}_{\ \ j} - \eta\xi^{*pi}_{\ \ j})(s'^{*j}_{\ \ i} - ce'^{*pj}_{\ \ i} - \eta\xi'^{*pj}_{\ \ i}) =$$

$$k_1[(s'^{*n}_{\ \ m} - ce'^{*pn}_{\ \ m})e^{*pm}_{\ \ n} +$$

$$(s^{*n}_{\ m} - ce^{*pn}_{\ \ m})e'^{*pm}_{\ \ n}], \quad k_1 = k(\varkappa_0)\frac{\partial k}{\partial\varkappa}\bigg|_{\varkappa=\varkappa_0}$$

For the metric tensor, strain tensor, and strain rate tensor

$$g_{ij}' = \nabla_i w_j' + \nabla_j w_i', \quad g'^{ij} = -(\nabla^i w'^j + \nabla^j w'^i), \quad \varepsilon_{ij}' = {}^1/_2 g_{ij}' \tag{1.10}$$

$$\xi_{ij}' = \frac{1}{2}\frac{\partial}{\partial t}(\nabla_i w_j' + \nabla_j w_i'), \quad \xi_j'^i = \frac{1}{2}\frac{\partial}{\partial t}(\nabla_j w'^i + \nabla^i w_j') + g_{jn}'\xi^{ni} + g'^{in}\xi_{nj}$$

For the velocity and acceleration vectors (the dot denotes the total time derivative)

$$v_i' = w_i^{\cdot\prime} + v_n\nabla_i w'^n, \qquad v_i^{\cdot\prime} = w_i^{\cdot\cdot\prime} + v_n^{\cdot}\nabla_i w'^n \tag{1.11}$$

The equilibrium equations and boundary conditions for the stress tensor increments are [10]

$$\nabla_n\sigma_i'^n - \sigma_n^m\nabla_i\nabla_m w'^n + \sigma_i^n\nabla_m\nabla_n w'^m + \rho' f_i + \rho f_i' = \rho w_i^{\cdot\cdot\prime} + \tag{1.12}$$

$$\rho v_n^{\cdot}\nabla_i w'^n + \rho' v_i^{\cdot}, \quad \sigma_i'^m n_m + \sigma_i^m n_m' = p_i'$$

Here ρ and f_i are the density and the mass force components, $\mathbf{n}'$ is the increment of the normal unit vector, p_i' is the increment in the surface load.

Let us limit ourselves to the case of developed plastic flow when the elastic strains can be neglected as small relative to the large plastic strain. This permits identification of the metric tensors $g^{\wedge}_{ij}$ and g^*_{ij} and all the quantities with corresponding indices. We henceforth omit the indices. Then, eliminating the quantities $\varepsilon'^{ei}_{\ j}$, $\xi'^{ei}_{\ j}$, $\xi'^{pi}_{\ j}$ and ψ', we obtain

$$(1 + \eta\psi)(\Sigma_j'^i - \lambda^*\Sigma_m'^m g_j^i) + \psi(2\mu + c)\sigma_j'^i + [2\lambda^*(1 + \eta\psi)\xi^{pi}_j - \tag{1.13}$$

$$\frac{1}{3}\psi(2\mu + c)g_j^i + 2\lambda^* c\psi e^{pi}_j\Big]\sigma_m'^m - 2[(1 + \eta\psi)\xi^{pn}_j + c\psi\varepsilon^{pn}_j]\sigma_n'^i +$$

$$\frac{2}{3}c\psi\varepsilon^{pn}_m\sigma_n'^m g_j^i + \frac{r_j^i}{k^2}\Big\{r_n^l(\lambda^*\Sigma_m'^m g_l^n - \Sigma_l'^n) +$$

$$2r_n^l\xi^{pm}_l(\sigma_m'^n - \lambda^*\sigma_k'^k g_m^n) + k_1\psi[(h_n^l - 2\mu e^{pl}_n)(\sigma_l'^n - \frac{1}{3}\sigma_k'^k g_l^n) +$$

$$2\lambda^* h_n^l e^{pn}_l\sigma_m'^m - 2h_n^l(\varepsilon^{pm}_l\sigma_m'^n - \frac{1}{3}\varepsilon^{pm}_k\sigma_m'^k g_l^n\Big)\Big]\Big\} =$$

$$2\mu\Big[(1 + \eta\psi)\xi_j'^i + c\psi e_j'^i - \frac{r_j^i}{k^2}(r_n^l\xi_l'^n - k_1\psi h_n^l e_l'^n)\Big]$$

$$\Sigma_j'^i = \frac{\partial}{\partial t}\sigma_j'^i + \xi_n'^i\sigma_j^n + \xi_n^i\sigma_j'^n - \sigma_n'^i\xi_j^n - \sigma_n^i\xi_j'^n$$

$$r_j^i = s_j^i - ce^{pi}_j - \eta\xi^{pi}_j, \qquad h_j^i = s_j^i - 2ce^{pi}_j, \qquad \lambda^* = \frac{\lambda}{3\lambda + 2\mu}$$

Here Σ'^i_j is the Jaumann derivative of the stress tensor increments. The remaining quantities are defined by (1.10).

Let us investigate the behavior of the perturbations in a short time interval in the neighborhood of the linearization point τ. We consider the coefficients of the linearized equations of state independent of time and defined at the instant $t = \tau$, we shall measure the time t for the perturbations from the instant τ. We shall consider the process of deformation stable if the perturbations damp out with time. This approach, understandably, does not afford the possibility of tracing the behavior of the perturbations for large values of time. An analogous plate and rod stability criterion under creep conditions was proposed in [11].

It has been shown in [12] that the effects of relaxation of the internal residual stresses, caused by the presence of an internal viscosity mechanism, can assure the possibility of plastic strain when the increment of the stress vector $\Delta\sigma$ either lies inside or outside the domain bounded by the loading surface of the preceding state. This permits the conclusion that there will be no unloading phenomenon for the medium model considered under small perturbations.

2. Let us seek the solution of (1.10) – (1.13) as

$$w_j'(X^i, t) = w_j(X^i)\, e^{st}, \qquad \sigma'^k_j(X^i, t) = t_j^k(X^i)\, e^{st} \tag{2.1}$$

(s is a complex quantity here).

Substituting (2.1) into (1.13) and selecting the concomitant coordinate system so that it agrees with the fixed Cartesian coordinate system x^α at the time of linearization, we obtain in the case of a triaxial precritical state of stress and strain

$$t_j^i = \delta_{ij} a_{im} b^{mn} w_{n,\,n} + (1 - \delta_{ij})\, c_{ij}\, (w_{i,\,j} + w_{j,\,i}) \tag{2.2}$$

Here

$$a_{im} = \frac{1}{\det \| d_{kl} \|} \frac{\partial}{\partial d_{mi}} \det \| d_{kl} \| \qquad (k,\ l = 1,\ 2,\ 3) \tag{2.3}$$

$$c_{ij} = \frac{(1 + \eta\psi)\,(2\mu + \sigma_i^{\ i} - \sigma_j^{\ j})\,(^1/_2\, s + \xi^{pi}_{\ i} - \xi^{pj}_{\ j}) + \mu c\psi\,(1 - 2\varepsilon^{pj}_{\ j})}{(1 + \eta\psi)\,(s + \xi^{pi}_{\ i} - 3\xi^{pj}_{\ j}) + \psi\,(2\mu + c - 2c\varepsilon^{pj}_{\ j})}$$

$$b^{ij} = 2\mu \left\{ \left[s\,(1 + \eta\psi) + (1 - 2\varepsilon^{pi}_{\ i}) \left(c + \frac{r_i^{\ i}}{k^2} k_1 h_j^{\ j} \right) \psi \right] \delta^{ij} - \right.$$
$$\left. \frac{1}{3} c\psi\,(1 - 2\varepsilon^{pj}_{\ j}) - \frac{r_i^{\ i}}{k^2} \left[s r_j^{\ j} + \frac{1}{3} k_1 \psi h_j^{\ j} (1 - 2\varepsilon^{pj}_{\ j}) \right] \right\}$$

$$d_{ij} = [s + (2\mu + c + s\eta)\,\psi - 2\,(1 + \eta\psi)\,\xi^{pi}_{\ i} - 2c\psi\varepsilon^{pi}_{\ i}]\,\delta_{ij} +$$
$$\lambda^* (1 + \eta\psi)(2\xi^{pi}_{\ i} - s) + 2c\psi \left(\lambda^* e^{pi}_{\ i} + \frac{1}{3} \varepsilon^{pj}_{\ j} \right) - \frac{1}{3} (2\mu + c)\,\psi +$$
$$\frac{r_i^{\ i}}{k^2} \{ r_j^{\ j} (2\xi^{pj}_{\ j} - s) - 2\lambda^* r_n^{\ n} \xi^{pn}_{\ n} + k_1\psi\, [h_j^{\ j} (1 - 2\varepsilon^{pj}_{\ j}) -$$
$$2\mu e^{pj}_{\ j} + 2\lambda^* e^{pn}_{\ n} h_n^{\ n}] \} \qquad (n = 1,\ 2,\ 3)$$

Let us consider the slow steady deformation of three-dimensional solids. We shall not take account of the inertial force in the original state. Substituting (2.2) into (1.12), we obtain a system of equations in terms of displacements

$$L_i^j w_j = 0 \qquad (i, j = 1, 2, 3) \tag{2.4}$$

$$L_i^j = \delta_i^m a_{mn} b^{nj} \frac{\partial^2}{\partial x_m \partial x_j} + (1 - \delta_i^j)(c_{ji} + \sigma_i^i - \sigma_j^j) \frac{\partial^2}{\partial x_i \partial x_j} +$$

$$(1 - \delta_i^m) c_{mi} \delta_i^j \frac{\partial^2}{\partial x_m^2} - \rho (s^2 - 2s\xi^{pi}_i) \delta_i^j$$

The general solution of the system (2.4) can be represented as a linear combination of three solutions

$$w_i^{(j)} = \frac{\partial \det \| L_k^l \|}{\partial (L_j^i)} \Phi^{(j)} \tag{2.5}$$

The functions $\Phi^{(j)}$ are determined from the differential equation

$$\det \| L_k^l \| \Phi^{(j)} = 0 \tag{2.6}$$

We note that here results analogous to [13] can be obtained. Let us limit ourselves to the construction of a solution in the case of plain strain. Equation (2.6) becomes

$$\left[A \frac{\partial^4}{\partial x_1^4} + B \frac{\partial^4}{\partial x_1^2 \partial x_2^2} + C \frac{\partial^4}{\partial x_2^4} - \rho D \frac{\partial^2}{\partial x_1^2} - \rho F \frac{\partial^2}{\partial x_2^2} + \rho^2 K\right] \Phi = 0$$

where

$$A = c_{12} a_{1k} b^{k1}, \quad B = a_{2k} b^{k2} a_{1n} b^{n1} - a_{1k} b^{k2} (c_{12} - \sigma_1^1 + \sigma_2^2) -$$

$$a_{2k} b^{k1} (c_{21} + \sigma_1^1 - \sigma_2^2) + (\sigma_1^1 - \sigma_2^2)(c_{12} - c_{21} + \sigma_1^1 - \sigma_2^2)$$

$$C = c_{21} a_{2k} b^{k2}, \quad D = s[(s - 2\xi^{p1}_1) c_{12} + (s - 2\xi^{p2}_2) a_{1k} b^{k1}]$$

$$F = s[(s - 2\xi^{p1}_1) a_{2k} b^{k2} + (s - 2\xi^{p2}_2) c_{21}], \quad K = s^2 (s - 2\xi^{p1}_1)(s - 2\xi^{p2}_2)$$

$$(k, n = 1, 2, 3)$$

A periodic solution along the Ox_1-axis can be represented as

$$\Phi = C_m^n e^{k_m x_2} \sin(\gamma_1 x_1), \quad \gamma_1 = \frac{\pi n}{l}, \quad n = 1, 2, \ldots, \infty; \quad m = 1, 2, 3, 4 \tag{2.7}$$

$$k_m = \pm \left(\frac{1}{2C} [(B\gamma_1^2 + \rho F) \pm \{(B^2 - 4AC)\gamma_1^4 + \rho (BF - 4DC)\gamma_1^2 + \rho^2 (F^2 - 4CK)\}^{1/2}\right)^{1/2}$$

The boundary conditions for $x_1 = 0$, $x_1 = l$ $\sigma_1'^1 = 0$ and $w_2' = 0$ are satisfied automatically, which corresponds approximately to hinged support conditions. For a thin-walled plate these conditions go over exactly into the hinged support conditions.

It should be noted that in the case of a "dead" loading, the boundary value problem (1.12), (2.4) is self-adjoint and the buckling can occur according to the type of static instability [14].

3. Let us consider the stability of deformation under the plane strain conditions for a rectangular strip of an incompressible, elastic viscoplastic material under compression along the OX^1-axis of the "dead" load. We consider the strip infinitely long in the positive direction of the OX^2-axis with the load-free boundary $X^2 = 0$.

Assuming the concomitant coordinate system X^i agrees with the Cartesian coordinate system x^α at the instant $t = \tau$, we have from (1.1)

$$x^1 = \frac{r}{r(\tau)} X^1, \quad x^2 = \frac{r(\tau)}{r} X^2, \quad x^3 = X^3, \quad r(0) = 1$$

where $r = r(t)$ is the degree of compression in the direction of the OX^1-axis. The components of the precritical state are determined at the running instant τ for $k(\varkappa) =$ const , by the following relationships:

$$v^1 = \frac{r^{\cdot}}{r}x^1,\ v^2 = -\frac{r^{\cdot}}{r}x^2,\ v^3 = 0,\ \xi^{p1}_1 = \frac{r^{\cdot}}{r},\ \xi^{p2}_2 = -\frac{r^{\cdot}}{r} \tag{3.1}$$

$$\xi^{p3}_3 = 0,\quad \varepsilon^{p1}_1 = \frac{1}{2}\left(1 - \frac{1}{r^2}\right),\quad \varepsilon^{p2}_2 = \frac{1}{2}(1 - r^2),\quad \varepsilon^{p3}_3 = 0$$

$$\sigma^1_1 = \sqrt{2}\,k + 2c\varepsilon^{p1}_1 + 2\eta\xi^{p1}_1,\ \sigma_2{}^2 = 0,\ \sigma_3{}^3 = \frac{k}{\sqrt{2}} + c\varepsilon^{p1}_1 + \eta\xi^{p1}_1$$

$$\varepsilon^{pi}_j = \xi^{pi}_j = \sigma_j{}^i = 0, \qquad i \neq j$$

According to (1.12), the boundary conditions on the free surface $x^2 = 0$ become

$$\sigma'^2_2 = 0, \qquad \sigma'^2_1 = 0 \tag{3.2}$$

Distinct types of instability follow from the solution (2.7) taking account of (2.5), depending on the values of the roots k_m. If all the values of the roots k_m are real, then a solution is possible which damps out with depth from the free surface $x^2 = 0$, i.e. the surface instability phenomenon holds [15]. If all the values of the roots are imaginary, then a solution is possible which is periodic along the Ox^2-axis, i.e. the internal instability phenomenon holds [16]. In the case of complex roots, the perturbations in the displacements will damp out with depth according to the internal instability type. Evidently different types of instability can be observed simultaneously for diverse combinations of the roots. Substituting the solution (2.7) into the boundary conditions (3.2), and hence taking account of (2.2), (2.3), (2.5), (3.1), we obtain the characteristic equations as a function of the kind of roots as a result of ordinary calculations.

Results of solving the characteristic equations on the "BESM-4" electronic digital computer showed that only surface instabilities can originate in metals. The dependence of the magnitude of the critical degree of compression r_* on the hardening coefficient $0.1 \leqslant c_0 \leqslant 1$ is shown in Fig. 1 for values of the yield point $-0.015 \leqslant k_0 \leqslant -0.003$, the coefficient of viscosity $0 \leqslant \eta_0 \leqslant 0.25$, and the strain rate $r^{\cdot} = -0.0001; -0.001$ ($c_0 = c / \mu$, $k_0 = k / \mu$, $\eta_0 = \eta / \mu$).

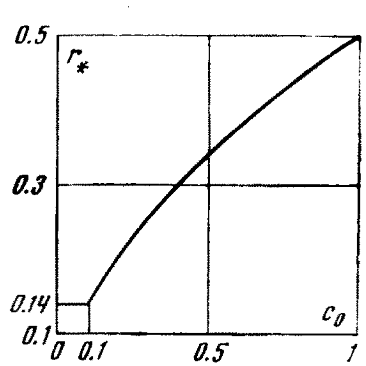

Fig. 1

Analysis showed that the influence of the yield point k_0 and the coefficient of viscosity η_0 within the above-mentioned limits on the magnitude of the critical strain is negligible. However, the numerical values of the critical loads hence obtained are unreal, therefore, the surface instability is not observed in practice. The surface instability phenomenon does not originate for slightly hardening materials ($c_0 < 0.1$) with the same values of k_0, η_0, r .

REFERENCES

1. Zahorski, S., On plastic instability in some cases of simple flow. Bull. Acad. Polon. Sci., Cl. IV, Ser. Sci., Techn., Vol. 12, № 11, 1964.

2. Zahorski, S., Kinematic stability in the case of slow steady plastic flow. Arch.

Mech. Stosowanej, Vol. 16, № 6, 1964.
3. Il'iushin, A. A., Deformation of a visco-plastic solid. Uch. Zap. Mosk. Univ., Vyp. 39, 1940.
4. Ishlinskii, A. Iu., On the stability of visco-plastic flow in a circular plate. PMM Vol. 7, № 6, 1943.
5. Ishlinskii, A. Iu., On the stability of visco-plastic flow in a strip and a circular rod. PMM Vol. 7, № 2, 1943.
6. Sporykhin, A. N., On the strain stability of elastic visco-plastic solids. Prikl. Mekh. Tekhn. Fiz., № 4, 1967.
7. Sporykhin, A. N., On the equilibrium stability of an elastic visco-plastic medium. Prikl. Mekh. Tekhn. Fiz., № 5, 1970.
8. Sedov, L. I., Introduction to the Mechanics of a Continuous Medium. Fizmatgiz, Moscow, 1962.
9. Truesdell, C. and Toupin, R., The Classical Field Theories. In: Handbuch der Physik, Vol. 3/1, pp. 226-793, Springer Verlag, Berlin-Göttingen-Heidelberg, 1960.
10. Green, A. E., Rivlin, R. S. and Schield, R. T., General theory of a small elastic deformation superposed on finite elastic deformations. Proc. Roy. Soc. A, Vol. 211, № 1104, 1957.
11. Rabotnov, Iu. N. and Shesterikov, S. A., Stability of bars and plates under creep conditions. PMM Vol. 21, № 3, 1957.
12. Ivlev, D. D. and Bykovtsev, G. I., Theory of a Hardening Plastic Solid. "Nauka", Moscow, 1971.
13. Guz'A. E., Stability of Three-dimensional Deformable Bodies. "Naukova Dumka", Kiev, 1971.
14. Bolotin, V. V., Nonconservative Problems of the Theory of Elastic Stability. (English translation), Pergamon Press, Book № 10490, 1963.
15. Biot, M. A., Surface instability in finite anisotropic elasticity under initial stress. Proc. Roy. Soc. A Vol. 273, № 1354, 1963.
16. Biot, M. A., Internal buckling under initial stress in finite elasticity. Proc. Roy. Soc. A Vol. 273, № 1354, 1963.

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INFLUENCE OF A RIGID INCLUSION ON THE STRESS INTENSITY NEAR THE TIPS OF A CRACK

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The stress intensity factor in a plate containing a rigid circular inclusion is determined by reduction to an integral equation with a Cauchy kernel and finding its numerical solution.

1. An elastic medium occupies the whole ($z = x + iy$)-plane with a circular hole